

Quantitative Problem Solving in Applied Science, Natural Sciences and Commerce:

“Hands on, Heads up” Learning

Key Ideas

1. focus on concepts: to increase understanding, and as an organizational framework
2. learn material sequentially
3. look for the “spiral of learning”: repetition and expansion of basic concepts
4. develop a systematic, methodical approach: talk yourself through each step
5. compute accurately, and eventually... quickly
6. persist
7. get help when needed

What is “Problem Solving”?

A form of learning based on discovery: to solve the problem, you must both think and compute systematically.

It is different from “exercise solving”, in which past routines are applied to solve similar problems, or a “trial and error” approach is used to match correct formula for the problem.

A central idea in problem solving is the use of “concepts”, which are the fundamental general ideas on which other notions may be built. In any subject, there are usually only a few basic concepts (often expressed as formula), which are applied in a variety of ways or situations. For example, basic concepts include *limit of function* in math, and *t-test* in statistics, *Newton's 2nd Law* in physics, *mole* in chemistry, and *liability* in accounting. Identifying and deeply understanding key concepts, and developing an organizational structure to allow you to recall how they relate to each other are essential elements in expert problem solving

The “spiral of learning” occurs when basic concepts are used repeatedly to solve a variety of problems. Frequently re-visiting those basic concepts allows them to become firmly fixed in your long-term memory, where they can be quickly recalled and applied.

People learn in different ways, and have different preferred styles of relating to their world, seeking sensory input, making information meaningful, and patterns of learning. It is very helpful to understand your own preferred learning style, and use methods that both mesh with and expand your style. See the [Index of Learning Styles](http://www.ncsu.edu/felder-public/ILSpage.html), by Felder and Silverman. www.ncsu.edu/felder-public/ILSpage.html

What is YOUR Approach to Quantitative Problem Solving?

Awareness of your own attitudes and habits is a good starting point to see your strengths and areas to change. Complete the following questionnaire to see where you are at.

Self-Assessment of Problem Solving Attitudes and Skills

taken from D.R.Woods, "Problem-Based Learning", 1994

<u>Attribute</u>	<u>Does the "+" or "-" best describe you?</u>	<u>Assessment</u>
Awareness	+ can describe problem solving process - cannot distinguish between PS and exercise solving	
Variety of PS skills	+ can apply a variety of methods and hints - attempts a "one-step" solution, few alternatives	
Emphasis on accuracy	+ checks, rechecks, concern for accuracy - concern for speed, little checking	
Active	+ writes things, makes table, sketches, lists - thinks in head, doesn't keep track, stares	
Monitors, Reflects	+ assesses continually, asks "where is this going?" - just does something, no evaluation	
Organized, Systematic	+ plans, anticipates, has and uses systematic plan - trial & error, impulsive, "plug and chug"	
Flexible, Open	+ willing to discard, sees options, takes a risk - quickly fixes on few options, won't switch	
Critical thinking	+ objective, learns from others, critically assesses data - accepts without question, doesn't use past experience	
Welcomes challenge	+ sees disequilibrium as good, uncertainty OK - resists confusion, uncomfortable with change	
Use of time	+ most time spent in exploring, defining, planning stages - most time spent doing, calculating, writing	
Overall Approach	+ based on fundamentals, concepts, principles, goals - searches for sample solution, "cooks" to make it work	
Decision-Making	+ applies criteria, draws conclusions based on evidence - decides on "gut" reaction	

Strengths:

Areas to work on:

Characteristics of Expert Problem Solvers

1. Attitude Characteristics

- Optimistic: you can do it
- Confident: the problem really does have a reasonable, but perhaps difficult, solution
- Willing to persevere: you aim for a complete and well reasoned solution, not an immediate or superficial one
- Concern for accuracy in reading: you concentrate, re-read and paraphrase to increase understanding, and translate unfamiliar words or terms
- Concern for accuracy in thinking: you work at a moderate to slow pace initially, perform operations carefully, check answers periodically, and draw conclusions at the end not part way through.

2. Skill Characteristics

- Systematic approach: you have a plan to follow, which
 - i) reduces the panic
 - ii) allows you to monitor your thought processes
 - iii) helps isolate errors in logic or computation
- Sound knowledge of basic concepts, which you mentally organize so you can recall and apply them
- Computational skill, at a good speed
- Habit of vocalizing or “thinking aloud”: you talk yourself through all thoughts
 - i) how to start the problem
 - ii) steps to break problems into parts
 - iii) decisions
 - iv) analyses
 - v) conclusions
- Awareness of your own thought processes: What did I do/learn? How did I do/learn this? How effective was my process?

Typical Characteristics of Novice Problem Solvers

1. They don't believe that persistent analysis is essential, therefore their effort and motivation to persist is weak.
2. They are careless in their reasoning.
3. They don't break problem into component parts and go step-by-step, therefore there are errors in logic and computation.
4. They focus on individual details, and don't see how details relate to concepts. Therefore, every problem feels new...how overwhelming!
5. Formula-memorizing is the main strategy.
6. They get behind in their learning, and sequential learning is hampered.
7. They lose confidence in their ability to solve problems, due to lack of success.

Strategies to Improve Problem Solving Skills

1. Self-Management: Use of time and resources

- Work on courses regularly: keep up so you can build on past knowledge (sequential learning), and get help quickly for difficulties.
- Do all the questions assigned, rather than dividing questions among group members, as you will get more practice with the concepts your Professor expects you to know. Aim for accuracy, then speed. Start assignments at least a week ahead of the due date, so you have time for help if needed.
- Use study groups to compare completed solutions to assigned problems. Teaching someone is a very effective learning and study technique.
- Choose problems wisely: learn to apply a specific concept to solve a variety of related problems. Start with simpler ones, and work up. Identify the relevant concept and practice until you know when and how to apply it, ie. you may not need to do all questions.
- Set a time limit: attempt a new problem every @ 15-20 minutes. If you can't complete a problem, check your "thinking strategies" and change to a new problem. Get help with the problems you couldn't complete, at tutorial, etc.
- Do some uncalculated solutions: If you are confident in your calculations- set up the solution but don't finish the calculation.
- Learn the necessary background and skills: find out from professor, course outline, etc. what the course involves and upgrade before the course begins if you don't feel confident about the prerequisites.
- Find and use help resources: use tutors, professors, TAs, friends, text, internet. For example: in accounting, economics, and finance texts, it is common to find examples that are quite similar to the problems at the end of the chapter. Work through the logic of the examples to develop a better understanding of how best to start the homework problems, if you run into trouble.

2. Thinking Skills: Organizational Frameworks

- Concept Summary Strategy

Learn and understand the small amount of information essential to each concept.

Concepts, ie. general organizing ideas, can be determined by:

- reading the learning objectives on the course outline or the course description,
- referring to the lecture outline to identify recurring themes,
- thinking about the common aspects of problems you are solving.

If in doubt, ask the professor what is important for you to "get".

See following description. See the video at <http://csd.mcmaster.ca/academic> (click Online Resources, scroll to "Math", select topic and format)

- General Problem-Solving Method

Follow a methodical, thorough approach to solve problems logically from first principles. See the description that follows.

- Decision Step Strategy

This strategy is a specific application of the General Method described above, suitable for use in statistics, accounting and other applied problem-solving situations. See description and example that follows. See the video at <<http://csd.mcmaster.ca/academic>> (click Online Resources, scroll to “Math”, select topic and format)

During the lecture or reading a text, focus on the process of solving the problem, not primarily on the computation. Identify the “decision steps” that lead to correct application of a concept. “Why did I move from this step to this step?”

- Expanded Thinking Strategy

Exams will challenge you to apply your knowledge to new situations, so prepare by creating questions or problems that are slightly different in some variable from your homework problems.

Actively think about the range of problems that are associated with a concept. Think in terms of both i) level of difficulty of the problems and ii) common kinds of difficult problems. Use this to anticipate different kinds of difficult problems for exam preparation, and solve some practice problems to test yourself.

Refer to the *Common Types of Difficult Problems* that follow for an example of expanded thinking, and see the video at <<http://csd.mcmaster.ca/academic>> (click Online Resources, scroll to “Math”, select topic and format)

- Homework Strategy

Use homework as a learning tool: prepare for the homework questions

-review class notes and understand the concepts in the examples. This might take 30 - 45 minutes.

-write the first line of a sample problem, close the book, and work as far as you can without looking.

-refer back to notes, and then again attempt sample

-repeat over again until you can solve the sample problem both accurately and quickly.

You will have memorized the rules in the process. This might take 1 hour.

-Start the homework questions.

-Interrogate your problem solutions: ask questions about the problem and your method of solving it. Eg.

1. What are the givens? Can the givens be classified as Assets, Liabilities, Owner's Equity, Income, Expenses, etc? Is there any Depreciation?
2. What is required?
3. Can I diagram this?
4. What concepts are referred to? Theorems? Operations?
5. Is the problem similar to others I solved/How?
6. What more do I need to understand this?
7. Are there any “tricks” to the question? If so, how do I deal with them?

-Keep track of problems you have trouble solving, isolate the particular difficulty, and get help to figure it out. Drill these problems until you are both accurate and fast in solving them.

Concept Summary Strategy

Taken from: Fleet, J., Goodchild, F. and Zajchowski, R., "Learning for Success", 2006
See <http://zackr.disted.camosun.bc.ca/> for a completed example. Click on SFU
QConference.

Purpose: to provide a structure for organizing fundamental, general ideas. The mental work involved in constructing the summary helps clarify the basic ideas and shift the information from working memory to long-term memory. This is an *excellent* study tool, for quick review.

Method:

The organizational elements are

- i) *Concept Title*
You can identify key ideas by referring to the course outline, chapter headings in the text, lecture outline. Sometimes concepts are thought of individually, other times they are meaningfully grouped for better recall. Eg. Depreciation, Capital Cost Allowance, and Half-Year Rule; acid, base and PH..
- ii) Use *general categories* to organize material, and then add *specific details* as appropriate. Sample general categories may include:
 - Allowable key formula- check summary page of text or ask professor
 - Definitions- define every term, unit and symbol
 - Additional important information- sign conventions, reference values, meaning of zero values, situations in which formula do not work, etc
 - Simple examples or explanations- use your own words, diagrams, or analogies to deepen your thinking and check your understanding
 - List of relevant knowns and unknowns- to help you know which concepts are associated with which problems, use crucial knowns to help distinguish among problems.

CONCEPT SUMMARY

Concept Title:

Allowable Key Formula:

Definitions of each symbol, and its units;

Additional important information: (eg. sign conventions, special characteristics, when concept doesn't work, special cases, etc)

Simple examples, explanations, cases:

Relevant knowns, and unknowns: (and words or phrases from word problems that signal these)

General Problem Solving Method

based on D.R. Woods, "Problem-based Learning", 1994

A systematic approach to problem solving helps the learner gain confidence, and is used consistently as a "blue print" by expert problem solvers as a way to be methodical, thorough and self-monitoring. This model is used in life generally, as well as in the sciences.

The steps are not linear, and multiple processes are happening in your brain simultaneously, but the basic template hinges on effective questioning as you carry out various steps

i. Engage

- Invest in the problem through reading about it and listening to the explanation of what is to be resolved. Your goal is to learn as much as you can about the problem before you begin to actually solve it, and to develop your curiosity (which is very motivating). Successful problem solvers spend two to three times longer doing this than unsuccessful problem solvers. Say "I want to solve this, and I can".

ii. Define the stated problem...a challenging and time consuming task

- Understand the problem as it is given you, ie. "What am I asked to do?"
- Ask "What are the givens? the situation? the context? the inputs? the knowns? etc.
- Determine the constraints on the inputs, the solution and the process you can use. For example, "you have until the end of class to hand this solution in" is a time constraint.
- Represent your thinking conceptually first, by reading the problem, drawing a pictorial or graphic representation or mindmap (see example attached), and then a relational representation.
- Then represent your thinking computationally, using a mathematical statement

iii. **Explore** and search for important links between what you have just defined as a problem, and your past experience with similar problems. You will create a personal mental image, trying to discover the "real" problem. Ultimately, you solve your "best mental representation" of the problem.

- Guestimate an answer or solution, and share your ideas of the problem with others for added perspective.
- Self-monitoring questions include: What is the simplest view? Have I included the pertinent issues? What am I trying to accomplish? Is there more I need to know for an appropriate understanding?

iv. **Plan** in an organized and systematic way

- Map the sub-problems
- List the data to be collected
- Note the hypotheses to be tested
- Self-monitoring questions include: What is the overall plan? Is it well structured? Why have I chosen those steps? Is there anything I don't understand? How can I tell if I'm on the right track?

v. **Do it**

- Self-monitoring questions include: Am I following my plan, or jumping to conclusions? Is this making sense?

vi. **Look back** and revise the plan as needed. Significant learning can occur in this stage, by creating other problems that use the same concepts, in anticipation for exams, and by evaluating your own thinking processes. This builds confidence in your problem solving abilities:-)

- Self-monitoring questions include: Is the solution reasonable? Is it accurate? (you will need to check your work to know this!) Does the solution answer the problem? How might I do this differently next time? How would I explain this to someone else? What other kinds of problems can I solve now, because of my success? If I was unsuccessful, what did I learn? Where did I go off track?

Decision Step Strategy: Specific Application of the General Method

Taken from: J. Fleet, F. Goodchild, R. Zajchowski, "Learning for Success", 2006
See <<http://zackr.disted.camosun.ca/>> for a completed example. Click on SFU Q
Conference

Purpose: to help learners focus on the process of solving problems, rather than on the mechanics of formula and calculations. The focus is on correct application of concepts to specific situations.

This strategy helps you to increase your awareness of the mental steps you make in problem solving, by "forcing" you to articulate your inner dialogue regarding procedure.

Method:

Identify the key decisions that determine what calculations to perform. In lecture, try to record the decision steps the professor uses but may not write down or post.

- i) Analyze solved examples, using brief statements focusing on steps you find difficult:
 - What was done in this step?
 - How was it done; what formula or guideline was followed?
 - Why was it done?
 - Any spots or traps to watch out for?

- ii) Test run the decision steps on a similar problem, and revise until the steps are complete and accurate.

Expanded Thinking Strategy: Common Types of Difficult Problems

Taken from: J. Fleet, F. Goodchild, R. Zajchowski, Learning for Success, 2006

Hidden knowns: needed information is hidden in a phrase or diagram Eg. “at rest” means initial $v = 0$ in physics.

Multipart-same concept: a problem may comprise 2 or more sub-problems, each involving the same concept. This type of problem can be solved only by identifying the given information in light of these sub-problems

Multipart-different concepts: same idea as above, but the sub-problems involve the use of different concepts

Multipart-simultaneous equations: same idea as above, but no single sub-problem can be solved by itself. You may have 2 unknowns and 2 equations or 3 unknowns and 3 equations, and you will need to solve them simultaneously, eg. using substitution, comparison, addition and subtraction, matrices, etc.

Work backwards: some problems look different because to solve them you have to work in reverse order from problems you have previously solved

Letters only: when known quantities are expressed in letters, problems can look different. If you follow the decision steps, they are not usually as difficult.

“Dummy variables”: sometimes a quantity that you think should be a known is not specified because it is not really needed- that is, it cancels out. Eg. mass in work-energy problems, temperature in gas-law problems.

Red herrings, unnecessary information: a problem may give you more information than is needed, which is confusing if you think you should use everything provided.

Resources

Online:

www.nscu.edu/felder-public/ILSpage.html, last accessed Nov. 2008. This is a free inventory, the Index of Learning Styles, to assess preferred learning styles, with additional information on interpretation of your profile

<http://csd.mcmaster.ca/academic>, last accessed Mar.2009. click "Online Resources", scroll to "Math", select topic and format

There are 3 videos on Problem Solving illustrating general ideas (Problem Solver I), differences in applying concepts vs. formula chasing (Problem Solver II), and applying the Decision Steps strategy (Problem Solver III).

<http://zackr.disted.camosun.bc.ca>, last accessed Mar. 2009. Click on SFU Q Conference. The personal web site for Richard Zajchowski, with examples of completed Concept Summaries, Decision Steps and other strategies

Books:

Fleet, J., Goodchild, F., Zajchowski, R. Learning for Success: Effective strategies for students, Thomson Nelson, 4th ed, 2006

Whimbey, A, Lockhead, J, Problem Solving & Comprehension, New Jersey: Lawrence Erlaum Associates, 5th ed., 1991

Woods, DR, Problem-based Learning: How to gain the most from PBL, Waterdown, ON: DR Woods, 1994